Primordial non-Gaussianities of gravitational waves beyond Horndeski

Tsutomu Kobayashi

Rikkyo University

Based on work with

Yuji Akita (Rikkyo University)

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See also Gao, TK, Yamaguchi, Yokoyama,
Phys. Rev. Lett. 107 (2011) 211301 [arXiv:1108.3513 [astro-ph.CO]]

Motivation

- $\langle \zeta\zeta\zeta \rangle$ has been studied extensively Maldacena (2003); Chen, *et al.* (2007); …
- Useful for discriminating different models of inflation
- Non-Gaussianity of primordial gravitational waves

$$\langle h_{ij} h_{ij} h_{ij} \rangle$$

- Probably extremely difficult to observe, but interesting

Maldacena (2003); Maldacena & Pimentel (2011); Soda, *et al.* (2011); Gao, *et al.* (2011); Cook & Sorbo (2013); Huang, *et al.* (2013); …

- This work: evaluate "$\langle h_{ij} h_{ij} h_{ij} \rangle$" in a general framework of inflation (= scalar-tensor theories beyond Horndeski)

Setup

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- **Horndeski theory:** the most general scalar-tensor theory with 2nd-order field equations

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R$$
$$+G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi, X)(\cdots)$$

$$X := -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Horndeski (1974); Deffayet, *et al*. (2011); TK, Yamaguchi, Yokoyama (2011)

—— free from the Ostrogradsky instability

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4 arbitrary functions

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—— free from the Ostrogradsky instability

- **Beyond Horndeski:** 2nd-order in time, but higher in space, thus can still avoid the Ostrogradsky instability

Gleyzes, Langlois, Piazza, Vernizzi (2015); Gao (2014)

Scalar-tensor theories beyond Horndeski

- Unitary gauge description of Horndeski theory

$$\mathcal{L} = G_2(\phi, X) + \cdots + G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + \cdots$$

$X = \dot{\phi}^2/2N^2$ $\quad\Big\downarrow\quad$ $\phi = \phi(t)$ $\quad$ ADM decomposition

$$\mathcal{L} = A_2(t, N) + \cdots + A_4(t, N)\left(K^2 - K_{ij}^2\right) + B_4(t, N)R^{(3)} + \cdots$$

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- This leads to spatially covariant theories of gravity

Gleyzes, Langlois, Piazza, Vernizzi (2015); Gao (2014)

$$\mathcal{L} = d_0(t, N) + d_1(t, N)R^{(3)} + d_2(R^{(3)})^2 + \cdots + d_4 a_i a^i + a_0 K + a_1 R^{(3)} K$$
$$+ a_2 R_{ij}^{(3)} K^{ij} + \cdots + b_1 K^2 + \cdots$$

$a_i := \partial_i \ln N$

Higher spatial derivatives, more general than Horndeski,
same sprit as EFT of inflation, Horava gravity, and ghost condensate

Cheung, *et al.* (2008) Horava, *et al.* (2009) Arkani-Hamed, *et al.* (2004)

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infinitely many arbitrary functions

Gleyzes, Langlois, Piazza, Vernizzi (2015); Gao (2014)

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XG3 (eXtended Galileon with 3-space covariance) proposed by Xian Gao

Gao (2014)

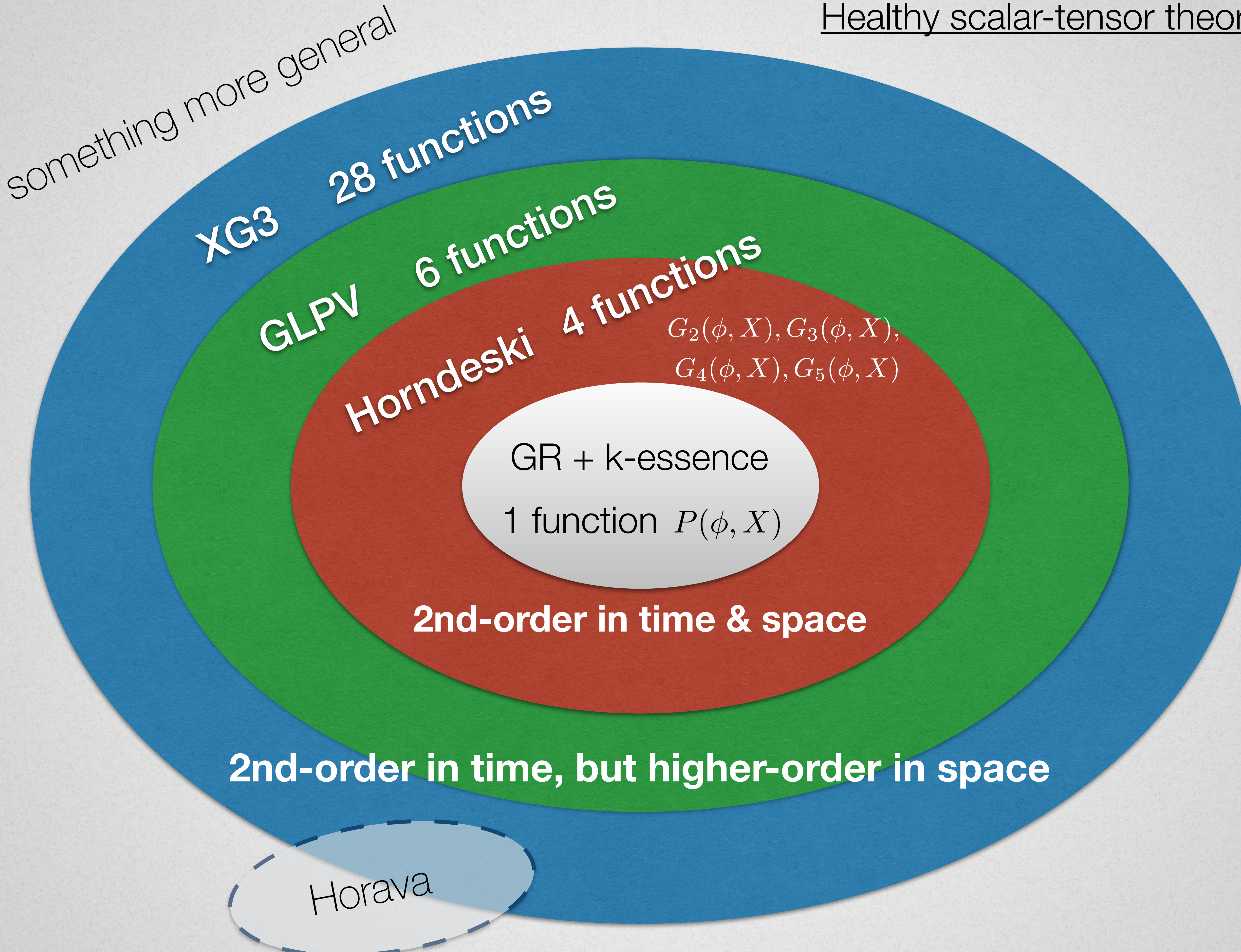
- 6 functions in GLPV … “minimal” extension
- 28 terms out of infinitely many possible combinations in XG3
  - (i) $\mathcal{L}$ contains derivatives of order 2 or less when going back to covariant form.

    $\longrightarrow$ ✗ $D^2 R^{(3)}, \left(D_i R^{(3)}_{jk}\right)^2$, ...
  - (ii) # of 2nd-order derivative operators does not exceed 3 when going back to covariant form.

    $\longrightarrow$ ✗ $(K_{ij})^4, \left(R^{(3)}_{ij}\right)^4$, ...
- The Lagrangian:

$$\mathcal{L} = \left[a_0(t,N) + a_1 R^{(3)} + \cdots\right] K + \left[a_2 R^{(3)}_{ij} + a_6 R^{(3)} R^{(3)}_{ij} + \cdots\right] K^{ij}$$
$$+ \cdots + c_1 K^3 + c_2 K K_{ij} K^{ij} + \cdots + d_0 + d_1 R^{(3)} + \cdots$$

Healthy scalar-tensor theories



Quadratic Lagrangian in XG3

GLPV, Horndeski

New term in **XG3**, Horava

$$\mathcal{L} = \frac{a^3}{8}\left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial_k h_{ij})^2 + 2\frac{d_3}{a^4}(\partial^2 h_{ij})^2\right]$$

$$\mathcal{G}_T := 2b_2$$

$$\mathcal{F}_T := 2d_1 + \dot{a}_2 + Ha_2$$

Modified dispersion relation

$$\omega^2 = c_h^2 k^2 + \epsilon^2 k^4 \eta^2, \quad c_h^2 := \mathcal{F}_T/\mathcal{G}_T, \ \epsilon^2 := -2H^2 d_3/\mathcal{G}_T$$

Analytic solution for *de Sitter, constant coefficients* Ashoorioon *et al.* (2011)

$$h_{\mathbf{k}} = \frac{\sqrt{2}}{a}\frac{\exp(-\pi c_h^2/8\epsilon)}{\sqrt{-\mathcal{G}_T \epsilon k^2 \eta}} W\left(\frac{i c_h^2}{4\epsilon}, \frac{3}{4}, -i\epsilon k^2 \eta^2\right)$$

W: Whittaker fn.

Cubic Lagrangian in XG3

GLPV, Horndeski

$$\frac{\mathcal{L}}{a^3} = \boxed{\frac{c_3}{8}\dot{h}_{ij}^3 + \frac{\mathcal{F}_T}{4a^2}\left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right)h_{ij,kl}}$$
$$+ \frac{a_7}{8a^4}\dot{h}_{ik}\partial^2 h_{jk}\partial^2 h_{ij} - \frac{b_6}{8a^2}\dot{h}_{ik}\dot{h}_{kj}\partial^2 h_{ij}$$
$$\boxed{- \frac{d_7}{8a^6}(\partial^2 h_{ij})^3 + \frac{d_3}{a^4}\partial^2 h_{ij}\left[\frac{1}{2}h_{ik,l}h_{jl,k} + h_{ij} \times \text{“}\partial\partial h\text{”}\right]}$$

New terms in **XG3**

Horava

Non-Gaussian amplitude

- Decompose h_{ij} in a helicity basis:

$$h_{ij}(\mathbf{k}) = \sum_{s=\pm} \xi^s(\mathbf{k}) \mathrm{e}_{ij}^{(s)}(\mathbf{k}) \qquad \xi^{\pm} = h^{+} \pm i h^{\times}$$

- In-in formalism Maldacena (2003)

$$\langle \xi^{s_1}(\mathbf{k}_1) \xi^{s_2}(\mathbf{k}_2) \xi^{s_3}(\mathbf{k}_3) \rangle = -i \int^{t} \mathrm{d}t' \langle [\xi^{s_1}(\mathbf{k}_1) \xi^{s_2}(\mathbf{k}_2) \xi^{s_3}(\mathbf{k}_3), H_{\mathrm{int}}(t')] \rangle$$

$$\langle \xi^{s_1}(\mathbf{k}_1) \xi^{s_2}(\mathbf{k}_2) \xi^{s_3}(\mathbf{k}_3) \rangle = (2\pi)^7 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{\mathcal{P}_h^2}{k_1^3 k_2^3 k_3^3} \mathcal{A}^{s_1 s_2 s_3}$$

Tensor non-Gaussianity in GR

$$\mathcal{L} = \frac{R}{2} + P(\phi, X) \quad \Rightarrow \quad \frac{\mathcal{L}}{a^3} = \frac{c_3}{8}\dot{h}_{ij}^3 + \boxed{\frac{\mathcal{F}_T}{4a^2}\left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right)h_{ij,kl}}$$

$$+\frac{a_7}{8a^4}\dot{h}_{ik}\partial^2 h_{jk}\partial^2 h_{ij} - \frac{b_6}{8a^2}\dot{h}_{ik}\dot{h}_{kj}\partial^2 h_{ij}$$

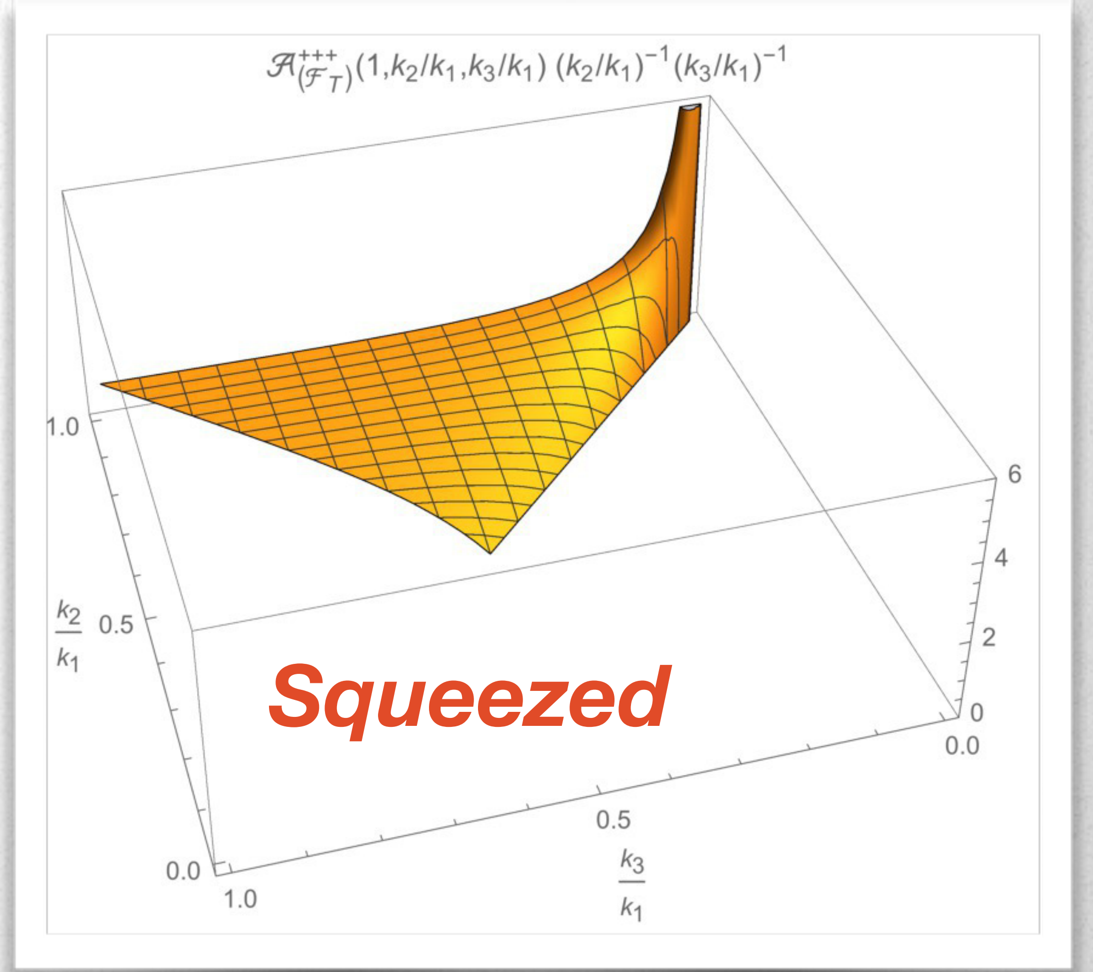
$$-\frac{d_7}{8a^6}(\partial^2 h_{ij})^3 + \frac{d_3}{a^4}\partial^2 h_{ij}\left[\frac{1}{2}h_{ik,l}h_{jl,k} + h_{ij} \times \text{“}\partial\partial h\text{”}\right]$$

$$\mathcal{A}^{+++} = \mathcal{A}^{+++}_{(\mathrm{GR})}$$
$$:= \frac{K_1}{16}\left(-1 + \frac{K_2}{K_1^2} + \frac{K_3}{K_1^3}\right)$$
$$\times \frac{K_1^8}{128K_3^2}\left(1 - 4\frac{K_2}{K_1^2} + 8\frac{K_3}{K_1^3}\right)$$

$K_1 := k_1 + k_2 + k_3$

$K_2 := k_1k_2 + k_2k_3 + k_3k_1$

$K_3 := k_1k_2k_3$



Tensor non-Gaussianity in Horndeski and GLPV

$$\frac{\mathcal{L}}{a^3} = \boxed{\frac{c_3}{8}\dot{h}_{ij}^3} + \frac{\mathcal{F}_T}{4a^2}\left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right)h_{ij,kl}$$

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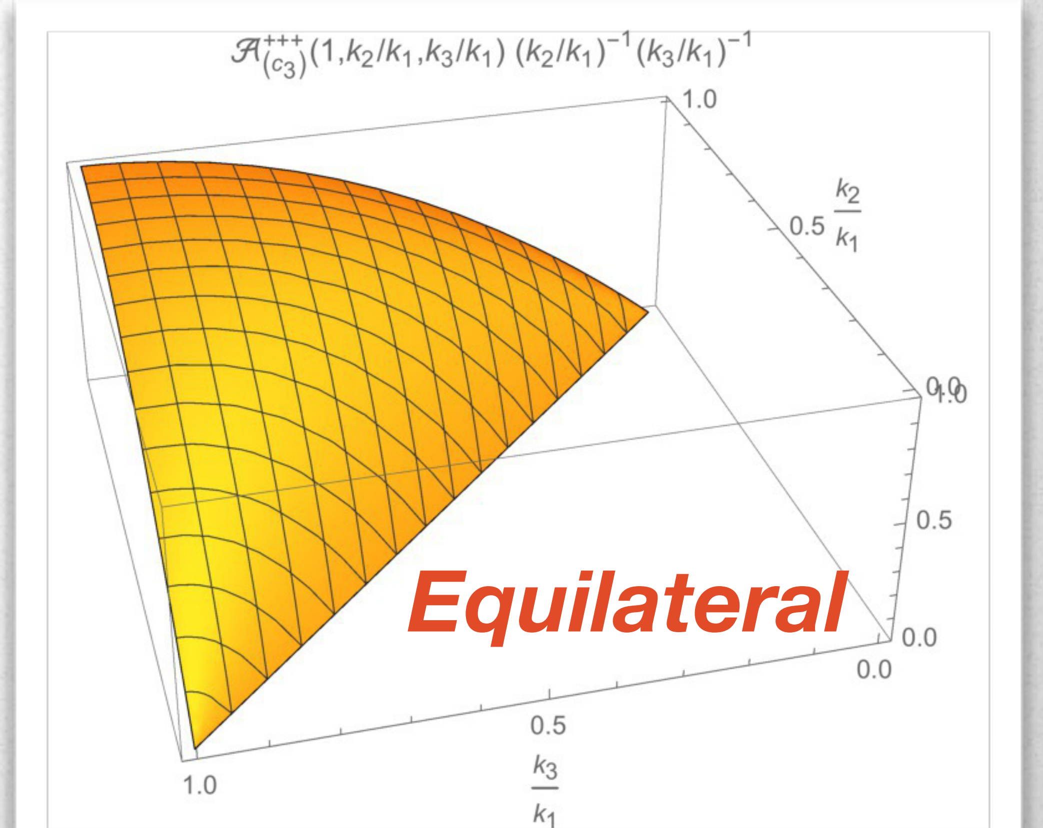
$$K_{ij}^3 \sim (\nabla_\mu \nabla_\nu \phi)^3 \subset \mathcal{L}_5$$

Only **2** independent terms

$$\mathcal{A}^{s_1 s_2 s_3} = \boxed{\mathcal{A}_{c_3}^{s_1 s_2 s_3}} + \mathcal{A}_{(\mathrm{GR})}^{s_1 s_2 s_3}$$

Equilateral *Squeezed*

Independent of functions,
same amplitude in all theories



Tensor non-Gaussianity in XG3

- Technical issue: mode function, not Hankel but Whittaker…
- Integration of (*Whittaker*)**3** cannot be performed analytically…

⇨ Assume modification to the dispersion relation is small

$$\mathcal{A}^{s_1 s_2 s_3} = \mathcal{A}_{(0)}^{s_1 s_2 s_3} + \frac{\epsilon^2}{c_h^4} \mathcal{C}^{s_1 s_2 s_3}$$

$\boxed{d_3 \text{ terms}}$ are assumed to be small

Ashoorioon *et al.* (2011)
for similar calculations of $\langle \zeta\zeta\zeta \rangle$

$$\mathcal{L} = \frac{a^3}{8}\left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial_k h_{ij})^2 + \boxed{2\frac{d_3}{a^4}(\partial^2 h_{ij})^2}\right]$$

$$\frac{\mathcal{L}}{a^3} = \frac{c_3}{8}\dot{h}_{ij}^3 + \frac{\mathcal{F}_T}{4a^2}\left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right)h_{ij,kl}$$
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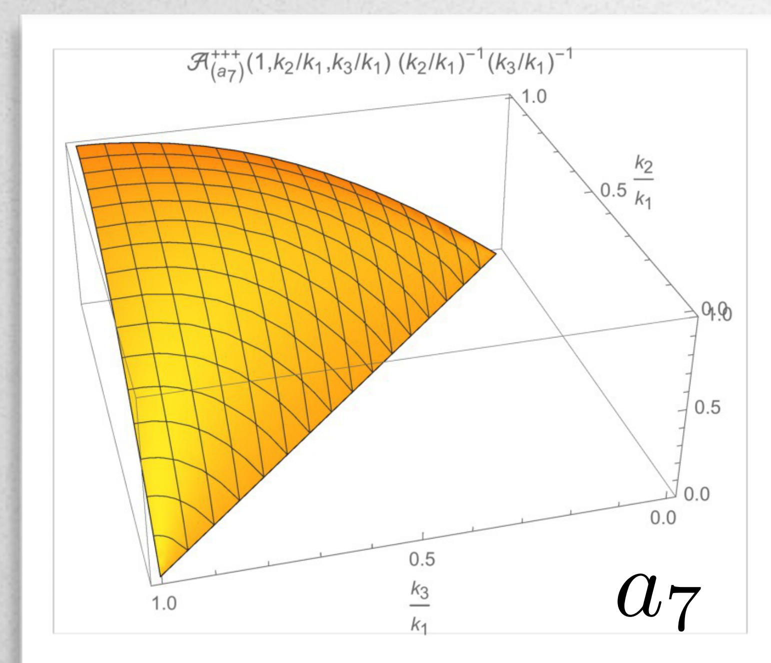
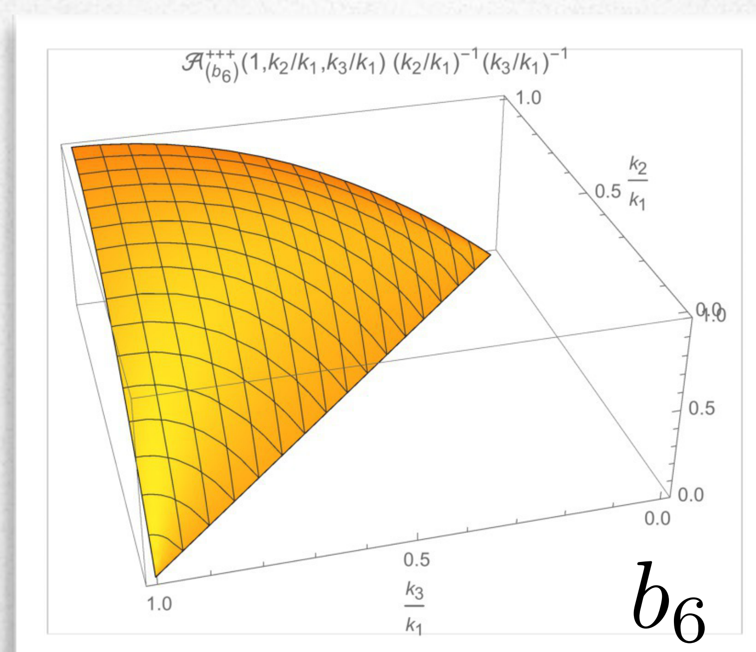
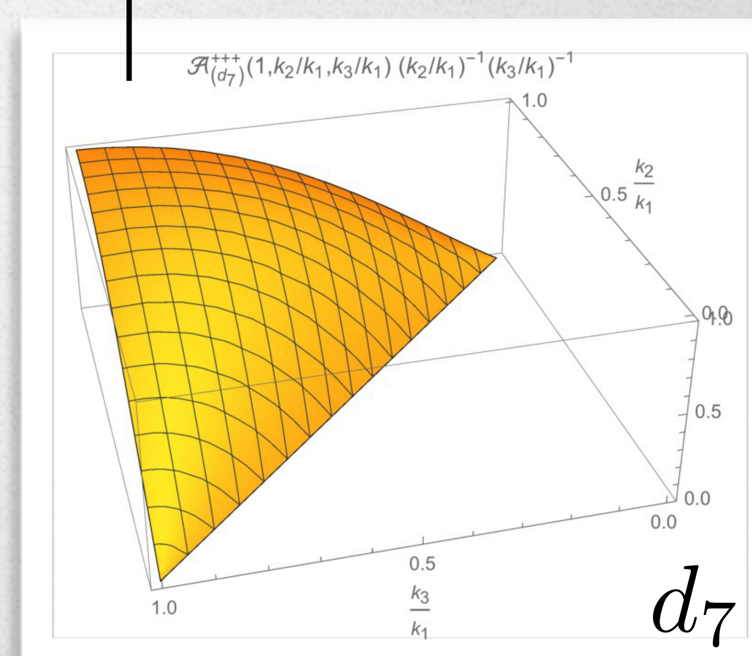
Horava gravity

Huang, *et al.* (2013)

Equilateral







Squeezed NG in XG3

- Only 2 terms generate squeezed non-Gaussianity

$$\frac{\mathcal{L}}{a^3} = \frac{c_3}{8}\dot{h}_{ij}^3 + \boxed{\frac{\mathcal{F}_T}{4a^2}\left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right)h_{ij,kl}}$$
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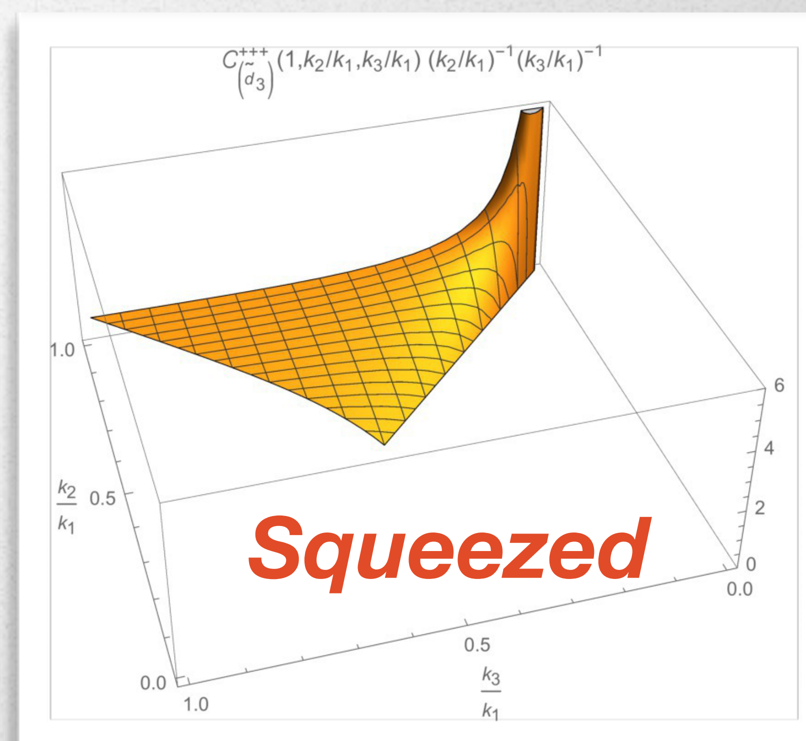
Small correction
(under our assumption)

Independent of functions,
same amplitude in all theories,
$\mathcal{A}^{s_1 s_2 s_3}_{(\mathrm{GR})}$



Summary

- Only 2 independent cubic terms in Horndeski *and GLPV*, one peaks at squeezed shape (=GR) and the other at equilateral shape
- 4 new terms in XG3
  - One is related to modified dispersion relation; this is assumed to be small in this work (due to technical reason)
  - The other 3 give equilateral non-Gaussianity
  - Squeezed non-Gaussianity $\simeq \mathcal{A}^{s_1 s_2 s_3}_{(\mathrm{GR})}$; independent of functions in the Lagrangian, same amplitude for all theories